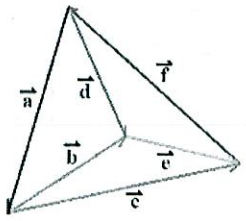
Write vector $\vec{f}$ in terms of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ in the diagram on the left. **SCORE: ____ / 2 PTS**



$\vec{f} = -\vec{c} - \vec{a}$

ALL ITEMS ON ALL QUESTIONS
① POINT
UNLESS OTHERWISE NOTED

Consider the vectors $\vec{f} = 3\vec{k} - 3\vec{j}$ and $\vec{g} = -\vec{i} + 2\vec{j} - \vec{k}$. **SCORE: ____ / 7 PTS**

$\langle 0, -3, 3 \rangle$ $\langle -1, 2, -1 \rangle$

[a] Find the angle between $\vec{f}$ and $\vec{g}$. (Your answer should be in radians.)

$$\cos^{-1}\frac{\vec{f}\cdot\vec{g}}{\|\vec{f}\|\|\vec{g}\|} = \cos^{-1}\frac{-6-3}{3\sqrt{2}\cdot\sqrt{6}} = \cos^{-1}\frac{-9}{6\sqrt{3}} = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

[b] If $\vec{e} = 7\vec{i} + c\vec{j} - 5\vec{k}$ is perpendicular to $\vec{g}$, find the value of c.

$\vec{e}\cdot\vec{g} = -7 + 2c + 5 = 0$

$c = 1$

Find the center and radius of the sphere $x^2 + y^2 + z^2 + 10x + 4y - 6z + 29 = 0$. **SCORE: ____ / 3 PTS**

$x^2 + 10x + 25 + y^2 + 4y + 4 + z^2 - 6z + 9 = -29 + 25 + 4 + 9$

$(x+5)^2 + (y+2)^2 + (z-3)^2 = 9$

CENTER $(-5, -2, 3)$ ($\frac{1}{2}$) RADIUS 3 ($\frac{1}{2}$)

Let P be the point $(-5, -2, 3)$. Let Q be the point $(3, 2, -1)$. Let R be the point $(-3, 4, -2)$. **SCORE: ____ / 18 PTS**

Let $\vec{w}$ be the vector with initial point Q and terminal point R.

[a] Find the equation of the sphere with P and Q as endpoints of a diameter.

CENTER = MIDPOINT = $\langle -1, 0, 1 \rangle$

RADIUS = $\frac{1}{2}$ DISTANCE = $\frac{1}{2}\sqrt{8^2+4^2+4^2} = 2\sqrt{6}$

$(x+1)^2 + y^2 + (z-1)^2 = 24$

(2)

[b] Find $\langle -1, 2, -1 \rangle \times \vec{w}$.

$\vec{w} = \overrightarrow{QR} = \langle -6, 2, -1 \rangle$

(3)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -1 \\ -6 & 2 & -1 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ -1 & 2 \\ -6 & 2 \end{matrix} = \begin{matrix} -2\vec{i} + 6\vec{j} - 2\vec{k} \\ +2\vec{i} - \vec{j} + 12\vec{k} \end{matrix} = \langle 0, 5, 10 \rangle$$

CHECK: $\langle 0, 5, 10 \rangle \cdot \langle -1, 2, -1 \rangle = 10 - 10 = 0$

$\langle 0, 5, 10 \rangle \cdot \langle -6, 2, -1 \rangle = 10 - 10 = 0$ ✓

[c] Find a unit vector in the opposite direction as $\vec{w}$.

$$-\frac{1}{\|\vec{w}\|}\vec{w} = -\frac{1}{\sqrt{41}}\langle -6, 2, -1 \rangle = \left\langle \frac{6\sqrt{41}}{41}, \frac{-2\sqrt{41}}{41}, \frac{\sqrt{41}}{41} \right\rangle$$

(2)

[d] If $\|\vec{v}\| = 3$, and the angle between $\vec{w}$ and $\vec{v}$ is $\frac{2\pi}{3}$ radians, find $\vec{w} \cdot \vec{v}$.

$$\|\vec{w}\| \|\vec{v}\| \cos\theta = \sqrt{41} \cdot 3 \cdot -\frac{1}{2} = -\frac{3\sqrt{41}}{2}$$

(2)

[e] In which octant is R ?

$O_{2+4} = O_6$